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Mean driving force in multichannel parallel-flow heat exchangers

TADEUSZ ZALESKI
Polish Academy of Sciences, Institute of Chemical Engineering, ul. Bałtycka 5, 44-100 Gliwice, Poland

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Abstract—Multichannel parallel-flow heat exchangers which are modelled assuming that the overall heat transfer coefficients and fluxes of heat capacities are independent of the temperature are termed 'linear-type exchangers'. The mean driving force defined for such exchangers has, in matrix notation, a form analogous to that for two-channel exchangers. It may be shown that for constant values of the coefficients it becomes an analogue of the logarithmic mean temperature difference.

1. INTRODUCTION

THE EQUATIONS presented are based, in principle, on a mathematical model discussed in refs. [1, 2]. Using a formal nomenclature, the model may be called a 'linear model with constant coefficients'. The words 'in principle' have been used to stress the fact that the first part of the analysis will also concern linear models with variable coefficients.

Thus, the assumptions corresponding to the parallel-flow heat exchanger analysed (Fig. 1) are as follows:

(1) The process is steady-state.

(2) Each fluid is ideally mixed in the direction perpendicular to the flow.

(3) The velocity profile is flat; the heat conductivity in the direction of the flow is negligible; the heat is transferred only in the direction normal to the axis of a channel.

(4) The channels are of equal length.

(5) The cross-section of each channel and, consequently, that of the exchanger is constant over the whole length l.

(6) Heat transfer coefficients, fluxes of heat capacities and thermal properties of the media are constant within each channel.

The differential balance of the heat exchanged between a medium flowing in a pth channel and all other media is

$$-\mathrm{d}Q_p = -W_p\,\mathrm{d}t_p = \sum_{\substack{j=1\\ j\neq p}}^{n} k_{pj}(t_p-t_j)\,\mathrm{d}f_{pj}$$

$$p = 1, 2, \ldots, n. \tag{1}$$

It should be noted that:

- the form of the linear model with variable coefficients is formally identical with (1);
- the heat transferred from a medium to its neighbours is taken as negative.

Introducing the dimensionless variable $z = x/l$ we may write equation (1) as follows:

$$-\mathrm{d}Q_p = -W_p\,\mathrm{d}t_p = \sum_{\substack{j=1\\ j\neq p}}^{n} k_{pj}\left(\frac{\mathrm{d}f_{pj}}{\mathrm{d}z}\right)(t_p-t_j)\,\mathrm{d}z. \tag{2}$$

It should be pointed out that the heat capacity flux, W_p, is positive when a medium flows in the direction of the Oz-axis, and negative ($W_p < 0$) with the flow in the opposite direction.

For comparison, the equations valid for the two-channel exchanger will be employed. Then, the notation may be simplified to:

$$Q_1 = Q;\, k_{12} = k_{21} = k;\, f_{12} = f_{21} = f \tag{3}$$

$$\frac{kf}{W_1} = a_1;\, \frac{kf}{W_2} = a_2;\, \delta = t_2 - t_1 \tag{4}$$

(obviously Q, k, f, W_1, W_2 and δ may be functions of z)

$$\mathrm{d}Q = -k\frac{\mathrm{d}f}{\mathrm{d}z}\delta\,\mathrm{d}z. \tag{5}$$

The total amount of heat exchanged is

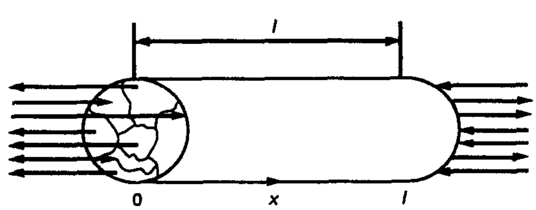

FIG. 1. Schematic representation of a multichannel parallel-flow heat exchanger. The channels in cross-sections $x = 0$ and 1 may be interconnected in any possible way.

NOMENCLATURE

$\mathbf{A}$	square matrix of the basic mathematical model, $\mathbf{W}^{-1}\mathbf{U} = [a_{ij}]_1^n$
a_{ij}	element of the matrix $\mathbf{A}$, $k_{ij}F_{ij}/W_i = u_{ij}/W_i$
$\mathbf{F}_p$	vectors of the heat transfer areas, $(F_{p1}, \ldots, F_{pn})$
$\mathbf{f}_p$	vectors of the heat transfer areas, $(f_{p1}, \ldots, f_{pn})$
F_{pj}, f_{pj}	area of heat transfer between the pth and jth media (channel), $j = 1, 2, \ldots, n$; $j \neq p$
$\mathbf{I}_m$	unit matrix of order m
$\mathbf{J}, \mathbf{J}_{n-1}$	canonical Jordan forms of matrices $\mathbf{A}$ and $\mathbf{P}$, respectively
$\mathbf{K}_p$	diagonal matrix of the overall heat transfer coefficients, $[k_{pj}]_{j=1}^{j=n}, j \neq p$
k_{pj}	overall heat transfer coefficients between the pth and jth channel lengths of the exchanger
$\mathbf{M}_p$	elementary matrix given by equation (A5) (see the Appendix)
$\mathbf{P}$	square matrix given by equation (A9) (see the Appendix)
$\mathbf{Q}$	'thermal' vector, $(Q_1, Q_2, \ldots, Q_n)$
Q_p	amount of heat exchanged between the pth medium and the remaining media
$\mathbf{S}, \mathbf{S}_p$	matrices leading to the canonical Jordan form
$\mathbf{T}_p$	fundamental matrix for equation (33)
$\mathbf{t}$	temperature vector, $(t_1, t_2, \ldots, t_n)$
t_i	temperature of medium i
t_{min}, t_{max}	lowest and highest temperatures in the exchanger
u_{ij}	$u_{ji} = k_{ij}F_{ij} = k_{ji}F_{ji}, j \neq i$
u_{ii}	$-\sum_{j=1, j\neq 1}^{n} u_{ij}$
$\mathbf{W}$	diagonal matrix of the heat capacity fluxes, $[W_i]_1^n$
W_i	flux of the heat capacity of medium i
x	linear coordinate
z	dimensionless coordinate, x/l.

Greek symbols

$\boldsymbol{\delta}_p$	vector of the temperature differences $(\delta_{p1}, \ldots, \delta_{pn})$
δ_{pj}	element of the vector $\boldsymbol{\delta}_p$, $t_p - t_j$; $j = 1, 2, \ldots, n; j \neq p$
$\boldsymbol{\theta}$	vector of dimensionless temperatures $(\theta_1, \theta_2, \ldots, \theta_n)$
θ_i	element of the vector θ, dimensionless temperature of medium i, $(t_i - t_{min})/(t_{max} - t_{min})$
λ_i	eigenvalues of $\mathbf{A}$ or $\mathbf{P}$.

$$Q = -\int_0^1 k\frac{df}{dz}\delta\, dz. \tag{6}$$

For a linear-type exchanger with constant coefficients

$$a_1, a_2, k = \text{const.}; f = Fz \tag{7}$$

and, consequently,

$$Q = -kF\int_0^1 \delta\, dz = -kF\delta_m. \tag{8}$$

Of course

$$\delta_m = \frac{\delta(1) - \delta(0)}{\ln\frac{\delta(1)}{\delta(0)}} = \delta_{ln} \quad \text{for}\, a_1 + a_2 \neq 0 \tag{9}$$

$$\delta_m = \delta(1) = \delta(0) = \delta = \text{const.} \quad \text{for}\, a_1 + a_2 = 0. \tag{10}$$

In this case the use of the logarithmic mean is not possible, as expression (9) becomes indeterminate. We may, however, employ the above concept by writing, for example

$$\delta(0) = \delta; \delta(1) = \varepsilon\delta\, (\varepsilon > 0, \varepsilon \neq 1) \tag{11}$$

and then calculating

$$\delta_m(\varepsilon) = \frac{(\varepsilon - 1)\delta}{\ln \varepsilon} \tag{12}$$

$$\delta_m = \lim_{\varepsilon \to 1} \delta_m(\varepsilon). \tag{13}$$

Naturally, the limit of this expression is δ.

2. OVERALL HEAT BALANCE FOR MULTICHANNEL EXCHANGERS

On introducing:

- the vector of the heat transfer area

$$\mathbf{f}_p = (f_{p,1}, \ldots, f_{p,p-1}, f_{p,p+1}, \ldots, f_{p,n}); \tag{14}$$

- the vector of the temperature difference

$$\boldsymbol{\delta}_p = (\delta_{p,1}, \ldots, \delta_{p,p-1}, \delta_{p,p+1}, \ldots, \delta_{p,n}) \tag{15}$$

where $\delta_{p,j} = t_p - t_j$; $j = 1, 2, \ldots, n$, $j \neq p$;

- the diagonal matrix of the overall heat transfer coefficients

$$\mathbf{K}_p = \{k_{p,1}, \ldots, k_{p,p-1}, k_{p,p+1}, \ldots, k_{p,n}\} \tag{16}$$

equation (1) may be written as

$$\mathrm{d}Q_p = -(\mathbf{K}_p\,\mathrm{d}\mathbf{f}_p)^{\mathrm{T}}\boldsymbol{\delta}_p \tag{17}$$

that is

$$\mathrm{d}Q_p = -\left(\mathbf{K}_p\frac{\mathrm{d}\mathbf{f}_p}{\mathrm{d}z}\right)^{\mathrm{T}}\boldsymbol{\delta}_p\,\mathrm{d}z. \tag{18}$$

The analogy between (18) and (5) is readily seen.

A formula describing the total amount of heat exchanged by the medium p is an analogue of equation (6)

$$\mathrm{d}Q_p = -\int_0^1\left(\mathbf{K}_p\frac{\mathrm{d}\mathbf{f}_p}{\mathrm{d}z}\right)\boldsymbol{\delta}_p\,\mathrm{d}z. \tag{19}$$

For a linear model with constant coefficients

$$f_{p,j} = F_{p,j}z;\mathbf{f}_p = \mathbf{F}_p z$$

$$F_{p,j} = \text{const.}, j = 1,2,\ldots,n, j \neq p \tag{20}$$

$$\mathbf{F}_p = (F_{p,1},\ldots,F_{p,p-1},F_{p,p+1},\ldots,F_{p,n}) \tag{21}$$

is a vector with constant coefficients.

As

$$k_{p,j} = \text{const.}, j = 1,2,\ldots,n, j \neq p \tag{22}$$

$\mathbf{K}_p$ is a matrix with constant coefficients, and

$$Q_p = -(\mathbf{K}_p\mathbf{F}_p)^{\mathrm{T}}\int_0^1 \boldsymbol{\delta}_p\,\mathrm{d}z \tag{23}$$

which, assuming

$$\int_0^1 \boldsymbol{\delta}_p\,\mathrm{d}z = (\boldsymbol{\delta}_p)_m \tag{24}$$

may be expressed as

$$Q_p = -(\mathbf{K}_p\mathbf{F}_p)^{\mathrm{T}}(\boldsymbol{\delta}_p)_m. \tag{25}$$

It may be seen that equations (23)–(25) are obtained by substituting the vector or matrix symbols for scalars in equation (8).

In the case analysed a simpler and more convenient method may be proposed for calculating the amount of heat exchanged between a given medium and all other media, employing directly the mathematical model of multichannel parallel-flow exchangers. If

$$\mathbf{Q} = (Q_1,Q_2,\ldots,Q_p,\ldots,Q_n) \tag{26}$$

is a vector with components Q_p, $p = 1,2,\ldots,n$;

$$\mathbf{t} = (t_1,t_2,\ldots,t_p,\ldots,t_n) \tag{27}$$

is a vector of the temperatures of the media, and

$$\mathbf{U} = [u_{ij}]_1^n \tag{28}$$

is a square matrix of dimension n, where

$$u_{ij} = k_{ij}F_{ij} \quad \text{for } i \neq j$$

$$u_{ii} = -\sum_{\substack{j=1\\ j\neq i}}^{n} k_{ij}F_{ij} \tag{29}$$

then

$$\mathbf{Q} = \mathbf{U}\int_0^1 \mathbf{t}\,\mathrm{d}z \tag{30}$$

$$Q_p = \mathbf{u}_{p\cdot}\int_0^1 \mathbf{t}\,\mathrm{d}z \tag{31}$$

where $\mathbf{u}_{p\cdot}$ is the pth row of the matrix $\mathbf{U}$. If the temperature profiles are known, the calculation of Q_p is straightforward. Equations (30) and (31) do not contain, however, the explicit forms of the driving forces equal to the temperature differences; from that point of view equations (17)–(19) and (23)–(25) are more interesting.

3. ANALOGUE OF A LOGARITHMIC MEAN DRIVING FORCE FOR MULTICHANNEL EXCHANGERS

In order to determine $(\boldsymbol{\delta}_p)_m$, equation (24), $\boldsymbol{\delta}_p$ has to be calculated as a function of z. A mathematical model with constant coefficients is a system of n ordinary differential equations, which should be solved for the functions of z defining the temperatures of the individual media:

$$\frac{\mathrm{d}\mathbf{t}}{\mathrm{d}z} = \mathbf{A}\mathbf{t}. \tag{32}$$

Appropriate transformations (see the Appendix) lead to a system of $n-1$ ordinary linear differential equations with constant coefficients, where the unknown functions correspond to the temperature differences between the pth medium and all other media:

$$\frac{\mathrm{d}\boldsymbol{\delta}_p}{\mathrm{d}z} = \mathbf{P}\boldsymbol{\delta}_p. \tag{33}$$

The set

$$\lambda_1,\lambda_2,\ldots,\lambda_{n-1},\lambda_n \tag{34}$$

is a spectrum of the matrix $\mathbf{A}$. The eigenvalues λ_i, $i = 1,2,\ldots,n-1$, n were arranged in (34) in such a way that for:

(a) $\sum_{i=1}^{n} W_i \neq 0$

$$\lambda_1,\lambda_2,\ldots,\lambda_{n-1} \neq 0;\lambda_n = 0; \tag{35}$$

(b) $\sum_{i=1}^{n} W_i = 0$

$$\lambda_1,\lambda_2,\ldots,\lambda_{n-2} \neq 0;\lambda_{n-1} = \lambda_n = 0. \tag{36}$$

Then, as is shown in the Appendix, the spectrum of the matrix $\mathbf{P}$ may be expressed as an ordered set

$$\lambda_1,\lambda_2,\ldots,\lambda_{n-1}. \tag{37}$$

Employing theorems of linear algebra it may be proved that there exists a non-singular square matrix $\mathbf{T}_p$ which satisfies equation (33). This matrix is called a fundamental (integral) matrix of equation (33). Thus

$$\frac{d\mathbf{T}_p}{dz} = \mathbf{P}\mathbf{T}_p. \quad (38)$$

It is usually assumed that the fundamental matrix is given by

$$\mathbf{T}_p = e^{\mathbf{P}z}. \quad (39)$$

Then

$$\mathbf{T}_p(z=0) = \mathbf{I} = \mathbf{T}_{p0}$$

$$\mathbf{T}_p(z=1) = e^{\mathbf{P}} = \mathbf{T}_{p1} \quad (40)$$

$$\mathbf{P} = \ln \mathbf{T}_{p1} = \ln(\mathbf{T}_{p1}\mathbf{T}_{p0}^{-1}). \quad (41)$$

Each vector solution of (33) may be written as

$$\boldsymbol{\delta}_p = \mathbf{T}_p\mathbf{c}_p \quad (42)$$

where $\mathbf{c}_p$ is a vector with any constant elements. If for $z = 0$ $\boldsymbol{\delta}_p(z=0) = \boldsymbol{\delta}_{p0}$, then

$$\mathbf{c}_p = \boldsymbol{\delta}_{p0}$$

$$\boldsymbol{\delta}_p = \mathbf{T}_p\boldsymbol{\delta}_{p0}. \quad (43)$$

According to (24) we have

$$(\boldsymbol{\delta}_p)_m = \left(\int_0^1 \mathbf{T}_p\, dz\right)\mathbf{c}_p.$$

(a) $\sum_{i=1}^{n} W_i \neq 0$

By virtue of (35) and (37), det $\mathbf{P} \neq 0$:

$$\int_0^1 \mathbf{T}_p\, dz = \mathbf{P}^{-1}[e^{\mathbf{P}} - \mathbf{I}]$$

$$= [\ln(T_{p1}T_{p0}^{-1})]^{-1}(\mathbf{T}_{p1} - \mathbf{T}_{p0}) = (\mathbf{T}_p)_{\ln}. \quad (44)$$

The above expression may be termed a 'matrix logarithmic mean'. Hence

$$(\boldsymbol{\delta}_p)_m = [\ln(\mathbf{T}_{p1}\mathbf{T}_{p0}^{-1})]^{-1}(\mathbf{T}_{p1} - \mathbf{T}_{p0})\mathbf{c}_p$$

$$= (\mathbf{T}_p)_{\ln}\mathbf{c}_p = [\ln(\mathbf{T}_{p1}\mathbf{T}_{p0}^{-1})]^{-1}(\boldsymbol{\delta}_{p1} - \boldsymbol{\delta}_{p0}) \quad (45)$$

where $\boldsymbol{\delta}_{p1} = \boldsymbol{\delta}_p\ (z=1)$.

A full analogy between (45) and (9) may be seen.

(b) $\sum_{i=1}^{n} W_i = 0$

In this case $(\boldsymbol{\delta}_p)_m$ cannot be expressed directly in terms of the matrix logarithmic mean, as from (36) and (37) it follows that the matrix $\mathbf{P} = \ln(\mathbf{T}_{p1}\mathbf{T}_{p0}^{-1})$ is singular. However, to obtain a formula corresponding to equation (45) we may write

$$\lambda_{n-1} = \lim_{\varepsilon \to 0} \varepsilon = 0 \quad (46)$$

$$(\delta_p(\varepsilon))_m = [\ln(\mathbf{T}_{p1}(\varepsilon)\mathbf{T}_{p0}^{-1})]^{-1}(\mathbf{T}_{p1}(\varepsilon) - \mathbf{T}_{p0})\mathbf{c}_p$$

$$= (\mathbf{T}_p(\varepsilon))_{\ln}\mathbf{c}_p = [\ln(\mathbf{T}_{p1}(\varepsilon)\mathbf{T}_{p0}^{-1})]^{-1}(\boldsymbol{\delta}_{p1}(\varepsilon) - \boldsymbol{\delta}_{p0}) \quad (47)$$

$$(\delta_p)_m = \lim_{\varepsilon \to 0}(\delta_p(\varepsilon))_m \quad (48)$$

4. SUMMARY

The heat balance was based on a mathematical model of multichannel parallel-flow heat exchangers. For practical calculations it is convenient to employ this model without any modifications.

In order to formulate a vector analogue of the logarithmic mean driving force it is necessary to transform the model into a form containing the temperature differences between an individual medium and all other media.

The assumption of the model with constant coefficients leads to a generalization of the concept of the logarithmic mean driving force. The final expression makes use of a formula defining the matrix logarithmic mean, and the equations obtained fully correspond to those valid for two-channel exchangers. However, while in the latter case the logarithmic mean temperature difference may actually be employed in computations, it remains basically an interesting theoretical generalization of the concept for the case of multichannel exchangers.

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APPENDIX

In refs. [1, 2] the differential model of multichannel parallel-flow heat exchangers was given as

$$\frac{d\boldsymbol{\theta}}{dz} = \mathbf{A}\theta \quad (A1)$$

where

$$\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_n)$$

$$\theta_i = \frac{t_i - t_{min}}{t_{max} - t_{min}}$$

(t_{min} and t_{max} are the temperatures of a cold and hot medium, respectively, at the inlet to the exchanger).

The matrix **A** is given by

$$\mathbf{A} = [a_{ij}]_1^n = \mathbf{W}^{-1}\mathbf{U}$$

where **W** is a diagonal matrix

$$\mathbf{W} = \{W_1, W_2, \ldots, W_n\}$$

$$\mathbf{U} = [u_{ij}]_1^n.$$

Obviously, if equation (A1) is satisfied by the vector $\boldsymbol{\theta}$, it is also satisfied by

$$\mathbf{t} = (t_1, t_2, \ldots, t_n) \quad (A2)$$

that is

$$\frac{d\mathbf{t}}{dz} = \mathbf{A}\mathbf{t}. \quad (A3)$$

The elements of the vector **t** are the temperatures of the media flowing within the channels of the exchanger. We would like, however, to find an equation satisfied by the following vector:

$$\boldsymbol{\delta}_p = (\delta_{p,1}, \ldots, \delta_{p,n}) \quad (A4)$$

where

$$\delta_{pj} = t_p - t_j, \quad j = 1, 2, \ldots, n; j \neq p. \tag{A4a}$$

Let $\mathbf{M}_p$ be an elementary matrix, which upon a pre-multiplication of any matrix (vector) moves its pth row into the position of the last row, and then adds it to the remaining rows taken with the opposite signs. It is convenient to write this matrix in block form:

$$\mathbf{M}_p = \left[\begin{array}{c|c|c} -\mathbf{I}_{p-1} & \begin{matrix}1\\ \vdots\\ 1\end{matrix} & \mathbf{0} \\ \hline \mathbf{0} & \begin{matrix}1\\ \vdots\\ 1\end{matrix} & -\mathbf{I}_{n-p} \\ \hline 0\ldots0 & 1 & 0\ldots0 \end{array}\right] \tag{A5}$$

$$\mathbf{M}_p^{-1} = \left[\begin{array}{c|c|c} -\mathbf{I}_{p-1} & \mathbf{0} & \begin{matrix}1\\ \vdots\\ 1\end{matrix} \\ \hline 0\ldots0 & 0\ldots0 & 1 \\ \hline \mathbf{0} & -\mathbf{I}_{n-p} & \begin{matrix}1\\ \vdots\\ 1\end{matrix} \end{array}\right]. \tag{A6}$$

Then

$$\mathbf{M}_p\mathbf{t} = (\delta_{p,1}, \ldots, \delta_{p,p-1}, \delta_{p,p+1}, \ldots, \delta_{p,n}, t_p) = \left[\begin{array}{c} \boldsymbol{\delta}_p \\ \hline t_p \end{array}\right]. \tag{A7}$$

Both sides of equation (A3) should be subjected to elementary transformations using $\mathbf{M}_p$:

$$\frac{\mathrm{d}(\mathbf{M}_p\mathbf{t})}{\mathrm{d}z} = (\mathbf{M}_p\mathbf{A}\mathbf{M}_p^{-1})(\mathbf{M}_p\mathbf{t}). \tag{A8}$$

Let

$$\mathbf{P} = \begin{bmatrix} (a_{1,1}-a_{p,1})(a_{1,2}-a_{p,2}) & \cdots & (a_{1,p-1}-a_{p,p-1})(a_{1,p+1}-a_{p,p+1}) & \cdots & (a_{1,n}-a_{p,n}) \\ (a_{2,1}-a_{p,1})(a_{2,2}-a_{p,2}) & \cdots & (a_{2,p-1}-a_{p,p-1})(a_{2,p+1}-a_{p,p+1}) & \cdots & (a_{2,n}-a_{p,n}) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ (a_{p-1,1}-a_{p,1})(a_{p-1,2}-a_{p,2}) & \cdots & (a_{p-1,p-1}-a_{p,p-1})(a_{p-1,p+1}-a_{p,p+1}) & \cdots & (a_{p-1,n}-a_{p,n}) \\ (a_{p+1,1}-a_{p,1})(a_{p+1,2}-a_{p,2}) & \cdots & (a_{p+1,p-1}-a_{p,p-1})(a_{p+1,p+1}-a_{p,p+1}) & \cdots & (a_{p+1,n}-a_{p,n}) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ (a_{n,1}-a_{p,1})(a_{n,2}-a_{p,2}) & \cdots & (a_{n,p-1}-a_{p,p-1})(a_{n,p+1}-a_{p,p+1}) & \cdots & (a_{n,n}-a_{p,n}) \end{bmatrix} \tag{A9}$$

(the matrix $\mathbf{P}$ is of order $n-1$), then the matrix $\mathbf{M}_p\mathbf{A}\mathbf{M}_p^{-1}$ may be expressed in block form as

$$\mathbf{M}_p\mathbf{A}\mathbf{M}_p^{-1} = \left[\begin{array}{c|c} \mathbf{P} & \begin{matrix}0\\ \vdots\\ 0\end{matrix} \\ \hline -a_{p,1}, -a_{p,2}, \ldots, \quad -a_{p,p-1}, \quad -a_{p,p+1}, \ldots, -a_{p,n} & 0 \end{array}\right] \tag{A10}$$

$$\frac{\mathrm{d}}{\mathrm{d}z}\left[\begin{array}{c} \boldsymbol{\delta}_p \\ \hline t_p \end{array}\right] = \left[\begin{array}{c|c} \mathbf{P} & \begin{matrix}0\\ \vdots\\ 0\end{matrix} \\ \hline -a_{p,1}, -a_{p,2}, \ldots, \quad -a_{p,p-1}, \quad -a_{p,p+1}, \ldots, -a_{p,n} & 0 \end{array}\right]\left[\begin{array}{c} \boldsymbol{\delta}_p \\ \hline t_p \end{array}\right]. \tag{A11}$$

Hence

$$\frac{\mathrm{d}\boldsymbol{\delta}_p}{\mathrm{d}z} = \mathbf{P}\boldsymbol{\delta}_p \tag{A12}$$

(the second of the above equations is identical with the pth equation of (A3)).

Let $\mathbf{J}$ be a canonical Jordan form of $\mathbf{A}$, while $\mathbf{S}$ is a matrix which transforms $\mathbf{A}$ into this form. Then

$$\mathbf{M}_p\mathbf{A}\mathbf{M}_p^{-1} = \mathbf{M}_p\mathbf{S}\mathbf{J}\mathbf{S}^{-1}\mathbf{M}_p^{-1} = (\mathbf{M}_p\mathbf{S})\mathbf{J}(\mathbf{M}_p\mathbf{S})^{-1}. \tag{A13}$$

It may be shown that

$$\mathbf{M}_p\mathbf{S} = \left[\begin{array}{c|c} \mathbf{S}_p & \begin{matrix}0\\ \vdots\\ 0\end{matrix} \\ \hline \mathbf{w}_p & 1 \end{array}\right] \tag{A14}$$

$$(\mathbf{M}_p\mathbf{S})^{-1} = \left[\begin{array}{c|c} \mathbf{S}_p^{-1} & \begin{matrix}0\\ \vdots\\ 0\end{matrix} \\ \hline \mathbf{v}_p & 1 \end{array}\right] \tag{A15}$$

where $\mathbf{S}_p$ is an easy to calculate, non-singular square matrix, while $\mathbf{w}_p$ and $\mathbf{v}_p$ are the row matrices whose elements are immaterial for further analysis.

On performing the operations indicated by equation (A13), employing (A14) and (A15) and comparing with (A10) we obtain

$$\mathbf{P} = \mathbf{S}_p\mathbf{J}_{n-1}\mathbf{S}_p^{-1} \tag{A16}$$

where $\mathbf{J}_{n-1}$ is a diagonal matrix and also a canonical Jordan form of $\mathbf{P}$

$$\mathbf{J}_{n-1} = \{\lambda_1, \lambda_2, \ldots, \lambda_{n-1}\}. \tag{A17}$$

Consequently, the solution of equation (A12) is straightforward.

FORCE MOTRICE MOYENNE DANS DES ECHANGEURS DE CHALEUR A ECOULEMENTS PARALLELES MULTICANAUX

Résumé—On appelle échangeurs de type linéaire, des échangeurs multicanaux à écoulements parallèles qui sont modélisés en supposant que les coefficients globaux de transfert thermique et les flux de capacités thermiques sont indépendants de la température. La force motrice moyenne définie pour ces échangeurs a, en notation matricielle, une forme analogue à celle pour les échangeurs à deux canaux. On montre que pour des valeurs constantes des coefficients elle est l'analogue de la moyenne logarithmique des températures.

MITTLERE TREIBENDE KRAFT IN MEHRKANAL-WÄRMEÜBERTRAGERN MIT PARALLELER STRÖMUNG

Zusammenfassung—Mehrkanal-Wärmeübertrager mit paralleler Strömung werden als "Wärmeübertrager von linearem Typ" bezeichnet, wenn bei ihrer Modellierung angenommen wird, daß die Gesamtwärmedurchgangs-Koeffizienten und die Wärmekapazitätsströme temperaturunabhängig sind. Die mittlere treibende Kraft, die man für solche Wärmeübertrager definiert, hat in Matrixschreibweise eine Form, die derjenigen für Zweikanal-Übertrager analog ist. Es kann gezeigt werden, daß für konstante Werte der Koeffizienten eine Entsprechung der mittleren logarithmischen Temperaturdifferenz entsteht.

СРЕДНЕЕ ЗНАЧЕНИЕ ДВИЖУЩЕЙ СИЛЫ В МНОГОКАНАЛЬНЫХ ПРЯМОТОЧНЫХ ТЕПЛООБМЕННИКАХ

Аннотация—Многоканальные прямоточные теплообменники в предположении, что коэффициенты суммарного теплопереноса и тепловые потоки не зависят от температуры, классифицируются как теплообменники линейного типа. Матричная запись среднего значения движущей силы рассматриваемых теплообменников сходна по форме со случаем двухканальных теплообменников. Показано, что при постоянных коэффициентах это значение становится аналогом логарифмической средней разности температур.